pressure equal to $-\sigma_{rp}$ at some radius, r, and an external pressure $-\sigma_{r_\rho}$ at ρ. Therefore, from Eq. (7)

$$\sigma_{rp} = \sigma_{r_{\rho'}} + \sigma_{r'_\rho} \tag{14}$$

where $\sigma_{rp'}$ equals $-P_{0'}$ which is the pressure required to produce full plastic flow in section considered.

From Eq. (4)

$$\sigma_{rp'} = 1.08\ \sigma_y \ln \frac{\rho}{r} \tag{15}$$

Substituting σ_{r_ρ} and $\sigma_{r_{\rho'}}$ from Eqs. (8) and (15), respectively, into Eq. (14) yields for the radial stress at radius r in the plastic region.

$$\sigma_{rp} = -\ \sigma_y\left[1.08 \ln \frac{\rho}{r} + \frac{b^2 - \rho^2}{\sqrt{(3b^4 + \rho^4)}}\right] \tag{16}$$

The well known equilibrium equation for a thick-wall cylinder is

$$\sigma_t = \sigma_r + r\,\frac{\mathrm{d}\sigma r}{\mathrm{d}r} \tag{17}$$

From Eqs. (16) and (17), the tangential stress in the plastic region becomes:

$$\sigma_{tp} = \sigma_y\left[-1.08 \ln \frac{\rho}{r} + 1.08 - \frac{b^2 - \rho^2}{\sqrt{(3b^4 + \rho^4)}}\right] \tag{18}$$

The use of the empirical coefficient 1.08 in Eqs. (16) and (18) leads to a very slight discontinuity in the σ_t distribution at the elastic–plastic interface. This is due to the coefficient varying somewhat with the elastic–plastic interface location. However, since the error is small, for the sake of simplicity it can be assumed that the coefficient is constant and independent of ρ.

To determine the strains in the plastic region, it is assumed that the only change in volume in the plastic region is elastic in nature and is given by:

$$\epsilon_r + \epsilon_t + \epsilon_z = \frac{1 - 2\mu}{E}(\sigma_r + \sigma_t + \sigma_z) \tag{19}$$

Defining ϵ_r and ϵ_t in terms of radial displacement (u) and substituting values of ϵ_z and σ_r and σ_t from Eqs. (13), (16) and (18) respectively ($\sigma_z = 0$) yields the following differential equation:

$$\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r} = \frac{\sigma_y}{E}\left\{(1 - 2\mu)\left[2.16 \ln \frac{r}{\rho} + 1{\cdot}08 - \frac{2(b^2 - \rho^2)}{\sqrt{(3b^4 + \rho^4)}}\right] + \frac{2\mu\rho^2}{\sqrt{(3b^4 + \rho^4)}}\right\} \tag{20}$$

This equation may be solved using the boundary condition of continuity of u at the elastic–plastic interface as given by Eq. (12). The resultant equation for displacement in the plastic region under pressure becomes:

$$\frac{u}{r} = \frac{\sigma_y}{E}\left[1.08\,(1 - 2\mu)\ln\frac{r}{\rho} + \frac{\rho^2(1-\mu) - b^2(1-2\mu) + (\rho^2 b^2)/(r^2)\,(2-\mu)}{\sqrt{(3b^4+\rho^4)}}\right] \quad (21)$$

Pressure-exterior Surface Strain

Graphs of exterior surface strain factor vs. pressure factor were obtained for all specimens tested. All curves for the same diameter ratio were averaged and are shown as the experimental curves in Fig. 9.

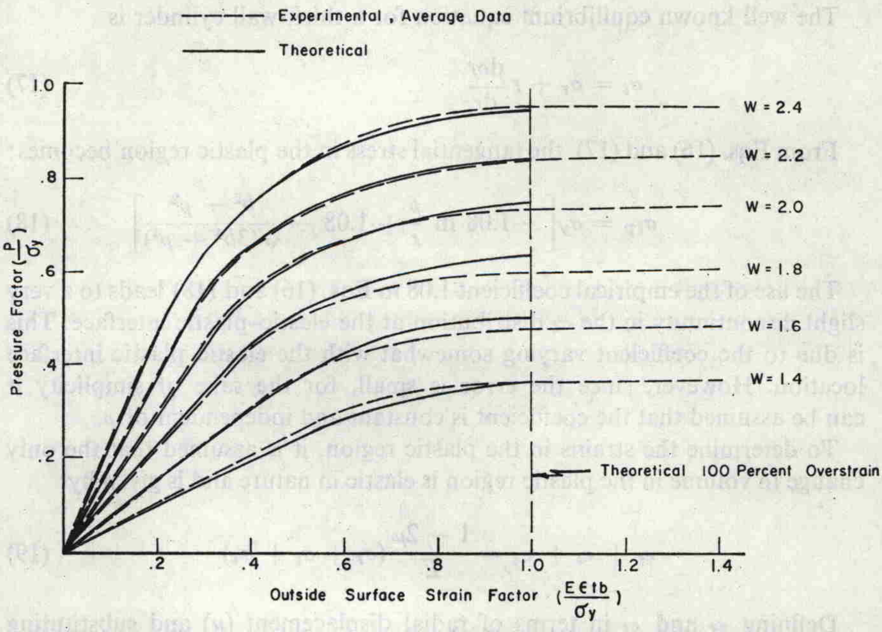

FIG. 9. Pressure factor vs. outside surface strain factor for various diameter ratios.

Since the radial stress at the bore ($r = a$) is equal to the internal pressure, the equation for P_ρ (pressure to produce plastic flow to a depth ρ) may be written from Eq. (16):

$$P\rho = \sigma_y\left[1.08\ln\frac{\rho}{a} + \frac{b^2 - \rho^2}{\sqrt{(3b^4+\rho^4)}}\right] \quad (22)$$